Chapter 9

2. $n = 100 \quad \mu = 100 \quad \sigma = 15$

a) A right tailed test $\quad z = \dfrac{x-\mu}{\sigma/\sqrt{n}}$

$H_0: \mu = 100$

$H_1: \mu > 100$

b) z-test would be done

4. Estimated standard error

a) $n = 9 \quad SS = 1.152$

$$s = \sqrt{SS/(n-1)} = \sqrt{1.152/8} = 0.3795$$

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{0.3795}{\sqrt{9}} = 0.1265$$

b) $n = 16, \; SS = 540$

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{540}{15}} = 6$$

$$SE_{\bar{x}} = s/\sqrt{n} = 6/\sqrt{16} = 1.5$$

c) $n = 25, \; SS = 600$

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{600}{24}} = 5$$

$$SE_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{5}{\sqrt{25}} = 1$$

6. 2, 18, 15, 5, 15, 8, 7

a) Mean $= \frac{1}{7}(2+18+15+5+15+8+7) = 10$

X	$X-\bar{X}$	$(X-\bar{X})^2$
2	-8	64
18	8	64
15	5	25
5	-5	25
15	5	25
8	-2	4
7	-3	9
		$\Sigma = 216$

$$S^2 = \text{Variance} = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{216}{7} = 30.86$$

b) $SE_x = \frac{s}{\sqrt{n}} = \frac{\sqrt{30.86}}{\sqrt{7}} = 2.0997$

8. $\alpha = 0.05$

a) $n = 6$, $d_f = n-1 = 6-1 = 5$

$t_{5,0.05} = \pm 2.5705$

b) $n = 12$, $d_f = n-1 = 12-1 = 11$

$t_{11,0.05} = \pm 2.201$

c) $n = 48$, $d_f = n-1 = 48-1 = 47$

$t_{47,0.05} = \pm 2.0117$

d (i) $t_{5,0.05} = 2.0150$

(ii) $t_{11,0.05} = 1.7959$

(iii) $t_{47,0.05} = 1.6\cancel{77}$

e) (i) $t_{5,0.01} = \pm 4.0317$

(ii) $t_{11,0.05} = \pm 3.1058$

(iii) $t_{47,0.01} = \pm 2.6846$

10. $n = 9$, $\mu = 20$

a) 43, 15, 37, 17, 29, 21, 25, 29, 27

$\bar{x}$ = Mean = $\frac{1}{9}(43+15+37+17+29+21+25+29+27) = 27$

x	$x-\bar{x}$	$(x-\bar{x})^2$
43	16	256
15	-12	144
37	10	100
17	-10	100
29	2	4
21	-6	36
25	-2	4
29	2	4
27	0	0
		$\Sigma(x-\bar{x})^2 = 648$

Variance $= \frac{\Sigma(x-\bar{x})^2}{n} = \frac{648}{9} = 72$

b) $-\mu + \bar{x} = -20 + 27 = +7$ the difference $= 7$

c) $s = \sqrt{s^2} = \sqrt{72} = 8.4853$

$SE_{\bar{x}} = s/\sqrt{n} = \frac{8.4853}{\sqrt{9}} = 2.8284$

d) $t_{9-1,\,0.05} = t_{8,\,0.05} = \pm 2.306$

$t_{stat} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{7}{2.8284} = \pm 2.4749$

The null hypothesis is rejected, therefore the treatment has a significant effect.

12. $n = 4$, $\mu = 35$, $M = 40.1$, $SS = 48$

$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{48}{3}} = 4$

(a) $\bar{x} - \mu = 40.1 - 35 = 5.1$

(b) $SE_{\bar{x}} = s/\sqrt{n} = 4/\sqrt{4} = 2$

c) $t_{n-1,\,\alpha} = t_{4-1,\,0.05} = t_{3,\,0.05} = \pm 3.1824$

$t_{stat} = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{40.1 - 35}{2} = \frac{5.1}{2} = \pm 2.55$

The null hypothesis is not rejected therefore no treatment effect.

14. $n = 6$, $\mu = 80$ $M = 92$

a) $s^2 = 54 \Rightarrow s = \sqrt{54} = 7.348$

$t_{stat} = \frac{M - \mu}{s/\sqrt{n}} = \frac{92 - 80}{7.348/\sqrt{6}} = \pm 4.0003$

$t_{n-1,\,\alpha} = t_{5,\,0.05} = \pm 2.5705$

H_0 is rejected and therefore the treatment has an effect.

b) $s^2 = 150 \Rightarrow s = \sqrt{150} = 12.25$

$$t_{stat} = \frac{M-\mu}{s/\sqrt{n}} = \frac{12}{12.25/\sqrt{6}} = \pm 2.3995$$

$$t_{5,0.05} = \pm 2.5075$$

H_0 is not rejected and therefore the treatment has no effect.

c) When the variance increases the t score reduces and therefore the treatment has no effect on the sample chosen.

16. 0.85 – blue

0.15 – Green

Cab in accident – Green 80% accuracy

Probability of a green car being in the accident $= 0.15 \times 0.8 = 0.12$

a) $M = 60.06$, $n = 16$

$\mu = 41$ $SS = 656.66$

$$s = \sqrt{\frac{SS}{n-1}} = \sqrt{\frac{656.66}{15}} = 6.616$$

$$t_{15,0.05} = \pm 2.1314$$

$$t_{stat} = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{60.06-41}{6.616/\sqrt{16}} = \pm 11.5236$$

Null hypothesis tests false and therefore, there is base rate fallacy

b) Measurements of effect size

$$\text{Cohen's } d = \frac{\bar{x}-\mu}{s} = \frac{60.06-41}{6.616} = 2.881$$

$$\text{odd ratio} = \frac{P_t(F)\,P_c(S)}{P_c(F)\,P_t(S)}$$ [Lcon

$$= \cancel{0.12(0.88)}$$

$$= \frac{0.88\,(0.41)}{(0.59)\,(0.12)} = \underline{\underline{5.096}}$$

$P_t(F)$ – probability of failure of treatment group

$P_c(S)$ – Probability of success of control group

$P_c(F)$ = " " failure " "

$P_t(S)$ = Probability of success of Treatment group.

18. $n = 8$, $\mu = 50$, $M = 55$

a) $s^2 = 32 \Rightarrow s = \sqrt{32} = 5.657$

$\alpha = 0.05$

Cohen's $d = \dfrac{M - \mu}{s} = \dfrac{55 - 50}{5.657} = 0.8839$

$$r^2 = \left(\frac{d}{\sqrt{d^2 + 4}}\right)^2$$

$$r^2 = \left[\frac{0.8839}{\sqrt{0.8839^2 + 4}}\right]^2 = 0.1634$$

Based on r^2, the effect is between small and medium

b) $\cancel{8/n} =$

$t_{7, 0.05} = \pm 2.3646$

$t_{stat} = \dfrac{M - \mu}{s/\sqrt{n}} = \dfrac{55 - 50}{5.657/\sqrt{8}} = \pm 2.499$

The null hypothesis tests false therefore there is a significant effect

b) $s^2 = 72 \Rightarrow s = \sqrt{72} = 8.485$

$t_{stat} = \dfrac{M - \mu}{s/\sqrt{n}} = \dfrac{5}{8.485/\sqrt{8}} = \pm 1.667$

$t_{7, 0.05} = \pm 2.3646$

The null hypothesis tests true and therefore no treatment effect.

Cohen's $d = \dfrac{M - \mu}{s} = \dfrac{5}{8.485} = 0.5892$

$$r^2 = \frac{0.5892}{\sqrt{0.5892^2 + 4}} = 0.2826$$

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c) When the variability increases the null hypothesis tests true and the effect size measures drop in magnitude.

20. $n = 12$

$\mu = 40$

$M = 37$ $\quad s^2 = 10.73$ $\quad \Rightarrow s = \sqrt{10.73} = 3.2757$

$$t_{stat} = \frac{M - \mu}{s/\sqrt{n}} = \frac{37 - 40}{3.2757/\sqrt{12}} = -3.1725$$

$t_{11, 0.05} = \pm 2.201$

The null hypothesis tests ~~true~~ false therefore ~~no~~ there is treatment effect.

b) Cohen's $d = \frac{M - \mu}{s} = \frac{+3}{3.2757} = 0.9158$

c) The null hypothesis is rejected and we conclude that there is ~~no~~ treatment effect. Our cohen's d confirms that the effect is large.

22. 2.08, 2.7, 3.42, 1.59, 2.04, 2.87, 3.36, 0.49, 3.82, 3.0

$$\text{Mean} = \frac{1}{10}(2.08 + 2.7 + 3.42 + 1.59 + 2.04 + 2.87 + 3.36 + 0.49 + 3.82 + 3.0)$$

Mean = 2.537

X	$X - \bar{X}$	$(X - \bar{X})^2$
2.08	-0.457	~~0.2088~~ 0.02656
2.7	0.163	
3.42	0.883	0.7797
1.59	-0.947	0.8968
2.04	-0.497	0.2470
2.87	0.333	0.1109
3.36	0.823	0.6773
0.49	-2.047	4.190
3.82	1.283	1.6461
3.0	0.463	0.2144
		$\Sigma = 8.9976$

$$s^2 = \frac{\Sigma(X - \bar{X})^2}{n} = \frac{8.9976}{10} = 0.89976$$

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$\alpha = 0.01$

$t_{9,0.01} = 2.8214$

$$t \text{ stat} = \frac{M - \mu}{s/\sqrt{n}} = \frac{2.537 - 0}{\sqrt{0.89976}/\sqrt{10}} = 8.46$$

The null hypothesis tests false and therefore individuals overestimate.

b) $$\text{cohen's } d = \frac{M - \mu}{s} = \frac{2.537}{\sqrt{0.89976}} = \cancel{2.8196}\ 2.6745$$

$$r = \frac{d}{\sqrt{d^2 + 4}} = \frac{2.\cancel{8196}\,6745}{\sqrt{\cancel{2.8196}\,2.6745^2 + 4}} = 0.8008$$

c)
$$\bar{x} \pm \left(t_{n-1,\alpha} \cdot \frac{s}{\sqrt{n}}\right)$$

$$2.537 \pm 2.8214 \times \frac{\cancel{0.89976}\ 0.9486}{\sqrt{10}}$$

$$2.537 \pm 0.8463 = (1.6907, 3.3833)$$